Energy Recovery in Miniature Trains

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INTRODUCTION

Electric motors powered by batteries are a popular power source for miniature trains. But the capacity of the batteries limits the time on task before they need to be swapped or the vehicle taken off task while they are charged.

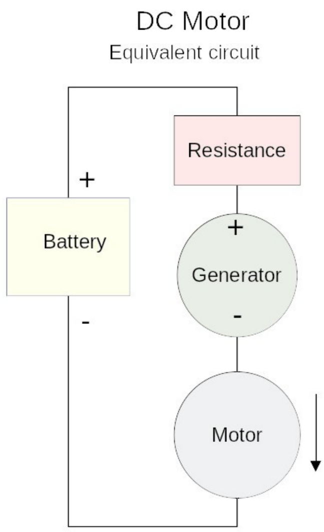
Kinetic energy recovery systems are now well understood and widely used, so the possibility of recovering kinetic energy from miniature trains seems a worthwhile ambition if it can extend their time on task.

This article explores the technical issues involved.

TORQUE, POWER AND GEARING

Direct Current (DC) electric motors, as commonly used in miniature locomotives, also act as generators when they are rotating. The figure shows a simplified equivalent circuit for a typical DC electric motor.

When a battery is connected to the motor, coil windings in the motor generate magnetic fields that interact with those of permanent magnets. This generates torque, and the maximum torque (in Newton-metres[1]) is one of the advertised parameters of the motor. The torque of the motor, which is proportional to the magnetic fields and therefore to the electrical current, acts through the drive system and wheels to provide the driving force of the train.



When the motor starts to rotate, the same windings interact with the permanent magnetic field and generate a “back EMF” voltage. This acts against the supply voltage, with the effect that the available voltage within the motor for generating the magnetic field is reduced, which leads to a reduction in current. So as the speed increases the torque correspondingly declines, even when the terminal voltage remains the same. This effect can be demonstrated easily by spinning a motor by hand while measuring the disconnected voltage on the terminals.

If the motor is left to spin unloaded at the rated voltage (another advertised parameter of the motor), the speed increases to a certain limit and goes no faster. At this point the back EMF almost matches the supply voltage, so the current and the torque are reduced to almost nothing and it stops accelerating. This speed (usually quoted in rpm) is another advertised parameter of the motor. As almost no mechanical work is being done (other than losses due to bearing viscosity, gears and aerodynamics) it requires almost no electrical power to drive it.

Clearly, when a motor is running at near it’s top speed it would not be producing the torque necessary to sustain the speed of a train, so when deciding on the gearing from the motor to the wheels of a locomotive the top speed of the motor is not a directly useful parameter to use.

Something to be aware of is the danger of powering a motor when it is stalled (i.e. prevented from rotating). In this situation the battery current is limited only by the electrical resistance of the windings, which is usually quite small. This results in a lot of heat being generated in the motor, which is not usually designed to sustain it for more than a brief period and will overheat and burn out. Therefore, when driving an electrically powered train it is important to introduce power progressively as it starts to move, especially when there is a heavy load and acceleration is slow.

According to any engineer’s greatest mentor, Isaac Newton, work is only done when a force is taken through a distance. So when a motor is stalled it is not delivering any mechanical work or power, even though it may be generating a lot of heat. At the other limit, it is not producing any torque when running at it’s top speed, so it is not delivering any useful power then either. But when the circuit is analysed[2] it is found that the maximum available mechanical power happens when the motor is running at half it’s rated speed. The electrical power at this point is is also one of the advertised parameters of the motor.

This electrical power is not the same as the delivered mechanical power as a significant amount is lost in the electrical resistance of the motor. The maximum mechanical power is calculated from the torque (half the stall torque) and the rpm at that speed (half the rated maximum).

1 May also be quoted in kilograms where 1kg=9.81Newtons.
2 For those who are interested, the maths are detailed in a later section.

Note: the quoted power rating of a motor is usually the power consumption at maximum mechanical output, which would be at half speed. The maximum electrical power actually occurs when the motor is (hopefully briefly) stalled on full power when stationary. So the controller and fuses need to be able to cope (for short periods) with up to twice the power than that usually specified for the motor.

To sustain a maximum speed, especially on an uphill incline, it therefore makes sense to gear the locomotive so that the motor is producing it's greatest power at the greatest speed that is required (or permitted). However, where there are steep inclines it may be necessary to gear the locomotive to provide at least the minimum amount of torque to sustain a minimum speed with the anticipated load, which may require a higher reduction gearing[3].

So when considering what motor to use, the following published parameters need to be considered:

- The maximum voltage (this is commonly 24V for the smaller gauges).
- The maximum torque (usually in Newton-metres, but may be kg-cm for small motors)[4].
- The unloaded speed at the rated voltage (in rpm).
- The maximum electrical power (usually in Watts).

To analyse the efficiency of a motor under various conditions, and to determine the recoverable energy from regenerative braking the electrical resistance is important to know. This is not usually quoted by the suppliers but it can easily be calculated as follows if the motor is not available for direct measurement:

- The current at peak power is simply the supply voltage divided by the rated power in Watts.
- The current when stalled on full power is usually double that.
- The internal resistance is the supply voltage divided by the stall current.

If you want to know this value you can calculate it from the motor specification without having to buy the motor to measure it for yourself.

Points to note

- The advertised torque usually only applies when it is stalled.
- The motor must not be sustained on high power while stalled.
- The torque reduces as the speed increases.
- The torque becomes zero at the rated top speed of the motor.
- The maximum power occurs at half the rated top speed of the motor.
- It is a popular misconception that electric motors do not benefit from gearboxes. The energy efficiency can be improved by changing the drive gearing to maintain high motor revolutions at low vehicle speeds.

3 This is detailed in a later section.
4 The torque may be specified as "stall torque".

ANALYSIS

The following shows how the torque of a motor varies with speed, and how to calculate the mechanical torque from the parameters usually provided for motors.

The following variables are used (those marked * are usually specified by the supplier):

Vs	* the supply voltage
Vb	the back EMF generated when in motion (Volts)
I	the current (Amps)
Im	the maximum (stall) current
Res	the circuit resistance (usually dominated by the motor) (Ohms)
R	the revolutions per minute
Rm	* the maximum (free-running) rpm
W	the electrical power (Watts)
Wm	* the electrical power at maximum mechanical power
Wr	power lost as heat in resistance
T	the torque (Newton-metres)
Ta	the torque per Ampere
Tm	* the stall torque
Wr	the mechanical work done per revolution
M	the mechanical power (work per second)
Mm	the maximum mechanical power
η	the power efficiency

If the reader is not interested in the workings, a summary of the equations most likely to be useful when designing for any particular motor is given at the end of this section.

Torque and mechanical power

Torque is the force (Newtons) at the rim of a wheel of radius 1 metre. So the work done when the wheel is rotated by one revolution is given by force * distance

1. Wr=T*2pi

and the mechanical power is force * distance-per-second, so from 1

2. M=Wr*revolutions-per-second = Wr*R/60 = T*2pi*R/60

2a (rearranging) M = pi*R*T/30

The torque presented at the driving wheels is magnified by a reduction gear or drive system, so the driving force for the vehicle is increased for driving wheels smaller than 1 metre in radius, in inverse proportion to their size. But the power is not changed by gearing, so smaller wheels or greater gear reduction will enable greater loads to be drawn up inclines, but the speed of doing so will be reduced.

Currents and voltages

The electrical current is the difference between the supplied voltage and back EMF, divided by the resistance

3 I=(Vs-Vb)/Res

But the back EMF is proportional to the rpm, and matches the supply voltage at maximum rpm

4 Vb=Vs*(R/Rm)

So, from 3 and 4 the current is the unopposed voltage divided by the resistance

5 I=(Vs-Vs*R/Rm)/Res = Vs*(1-R/Rm)/Res

Torque, current and speed

The torque is proportional to the magnetic field, which is proportional to the current so

6 $T=Ta*I$ where Ta (the torque per Ampere) is a constant determined by the design of the motor

so from 5 and 6 we can find the torque in terms of speed

7 $T=TaVs(1-R/Rm)/Res$

This is clearly at a maximum when R=0 (the motor is stalled), so it can be restated in known motor parameters

7a $T=Tm*(1-R/Rm)$

Mechanical power and speed

So from 2a and 7a we can calculate the mechanical power in terms of the speed

8 $M = pi*R*T/30 = pi*R*Tm*(1-R/Rm)/30$

8a (rearranging) $M = pi*Tm*(R-R^2/Rm)/30$

This is a quadratic equation which is inverted (hump upwards) that passes through zero when R is zero or when R=Rm. The peak mechanical power is exactly half way between these points, which is when R=Rm/2, so the peak mechanical power is

9 $Mm = pi*Tm*Rm/120$

Circuit resistance

In 6 we found that at half revolutions we have half the maximum torque, and therefore half the maximum current. So if we know the rated electrical power at maximum output we can calculate the current at half speed from the rated maximum power.

10 $I = Wm/Vs$

And therefore the current when stalled at full voltage must be twice that

11 $I = 2*Wm/Vs$

So the circuit resistance of the motor is Vs/Im or

12 $Res=Vs^2/(2*Wm)$

Current, power and torque per Ampere

The current at any speed can be calculated from 4

13 $I = Vs*(1-R/Rm)/(Vs^2/(2*Wm))$

13a (rearranging) $i = 2*Wm*(1-R/Rm)/Vs$

The electrical power can be calculated for any speed from the supply voltage and this current

14 $W = 2*Wm*(1-R/Rm)$

This can be seen to be maximum when stalled and zero when free running.

The torque per Ampere is found from the stalled torque and the stall current from 11

15 $Ta=Tm*Vs/(2*Wm)$

Efficiency (getting the most from the battery)

At any speed the efficiency can be calculated as the mechanical power (output) as a fraction of the electrical power (input).

The energy lost as heat in the circuit resistance is defined by the difference between the supply voltage and the back EMF, so the back EMF represents the share of the electrical power used by the motor.

16 $\eta = Vb/Vs$

and from 4

17 $\eta = R/Rm$

This is interesting because it shows that the mechanical efficiency is linearly related to the motor speed, and is (obviously) zero when stalled. It also shows the efficiency is theoretically 100% when free running, but as neither power nor work is involved that is perhaps not a very useful gem of information.

But it is of value to understand that a motor geared to run quickly at a required track speed will use battery power more efficiently than the same one geared to run more slowly at the same track speed. This implies that it is wasteful on battery power to gear a locomotive to be able to run a train faster than the intended (or permitted) speed.

More about torque

Some motors are sold without advertising the maximum torque. But this can be calculated from the above equations as follows.

From 17 the efficiency η at half revolutions is 50% (assuming insignificant mechanical losses). This corresponds with maximum output power, for which the rated electrical power is given as Wm. The mechanical power Mm at this speed is given from 9, which should therefore correspond with 50% of the electrical power. So the maximum torque (which is when the motor is stalled) can easily be found.

18 Wm/2 = Mm = pi*Tm*Rm/120

19 (rearranging) Tm = Wm*60/(pi*Rm)

This is calculated from the half-speed performance, so it may be optimistic depending on the mechanical losses of the motor at that speed.

If the torque is specified for a motor it can be compared with this calculation to see whether there are unadvertised inefficiencies in the motor.

As the available torque is proportional to efficiency, it may be useful to adjust it according to the efficiency of the drive train. So if the drive train is estimated as 80% efficient, then the available torque at the wheels could be corrected by a factor of 0.8 when calculating available power.

SUMMARY OF TORQUE AND POWER

The features most likely to be useful when designing a locomotive to use a particular DC motor can be expressed in terms of the parameters normally specified by the supplier:

Maximum (stall) electrical current	11	I = 2*Wm/Vs
Electrical current vs speed	13a	I = 2*Wm*(1-R/Rm)/Vs
Electrical power vs speed	14	W = 2*Wm*(1-R/Rm)
Electrical current at maximum output power	10	I = Wm/Vs
Circuit resistance	12	Res = $Vs^2/(2*Wm)$
Torque vs speed	7a	T = Tm*(1-R/Rm)
Mechanical power vs speed	8a	M = pi*Tm*R(1-R/Rm)/30
Maximum mechanical power	9	Mm = pi*Tm*Rm/120
Motor speed at maximum output power		R = Rm/2
Power efficiency	17	η = R/Rm
Theoretical maximum torque (if not advertised)	18	Tm = Wm*60/(pi*Rm)

Points to note in locomotive design

- The maximum current taken from the controller could be twice what you would expect from the rated maximum power of the motor.
- The greatest output power occurs at half the motor maximum (free running) speed.
- The electrical efficiency increases with motor speed, so gearing to support an unnecessarily high maximum train speed is wasteful on battery power.
- It is a popular misconception that electric motors do not benefit from gearboxes. The energy efficiency can be significantly improved by changing the drive gearing to maintain high motor revolutions at low vehicle speeds.
- The controller itself will consume some power, part of which is required by the electronics, and part due to switching resistance. This could be significant, but it may vary widely with the quality of controller designs. Some controllers get hot in normal use, which indicates a waste of battery power.
- All the above supposes the mechanical losses can be ignored. But the viscous losses in the bearings and aerodynamics increase substantially towards the higher speeds, especially for high-speed motors. This varies according to motor design and speed, but for optimum battery efficiency it may be best to keep clear of the top 15% or so of the motor speed.

Examples

A small 24V DC motor is rated at 120W with a maximum speed of 2800 rpm.
The torque and DC resistance are not quoted.

- The current at maximum power is therefore 5A so the stall current would be 10A.
- The DC resistance is therefore calculated as 2.4 Ohms, which corresponds to the measured value.
- The stall torque is unadvertised, but calculated as 0.82 N-m.
- At full voltage under no load the measured current is 0.33A which corresponds to 8W of dynamic losses, or 6.7% of the power.
- At half voltage under no load the measured current is 0.28A which corresponds to 3.36W of dynamic losses. This represents 5.6% of the available power, so the estimated torque can be adjusted downwards to 0.77 N-m
- In this case it seems that although the dynamic losses increase with speed they seem to average about 5.6% throughout the range, which implies it is mostly friction in the bearings or brushes.

REGENERATIVE POWER

It has been shown that when a DC motor is running it generates an internal voltage. When powered electrically this voltage opposes the supply, with the effect of reducing current and torque as the speed increases. But when the motor is being driven mechanically without an electrical circuit this internal voltage appears at the terminals of the motor. If this voltage is used to drive current into some sort of external load the power so derived is accounted for by reducing the vehicle kinetic energy.

So connecting some sort of electrical load to a motor when the vehicle is on overrun will cause a braking effect.

The generated voltage is the same as it would be when under power at the same speed. In other words, if the motor on overrun is driven to it’s specified maximum speed, then it will generate a voltage that matches the rated supply voltage.

Commonly, a motor controller will use transistor switches to connect a resistor, or even a short circuit, to the motor when power is cut off by the operator. With powerful motors this effect can be dramatic, evens dangerous, so it is usually mitigated by having PWM control so the load is connected part-time. By having a controllable duty cycle the braking effect and heat can be moderated but the use of mechanical brakes, and the associated wear and tear, can still be reduced.

If a resistive load is used then all the kinetic energy taken from the vehicle ends up as heat somewhere in the electrical system – either the external resistor (if there is one) or the motor itself. So a short-circuit, if sustained, could generate so much heat that the motor becomes damaged.

But it seems such a waste to throw away all that hard-earned kinetic energy as heat. Surely it must be possible to recover some of it to put some charge back into the battery. Preferably from the motor itself, so an additional piece of machinery is not required.

The following sections explore two different ways of collecting the regenerated energy so that some of it can be used to top up the charge in the battery and extend it’s time on task.

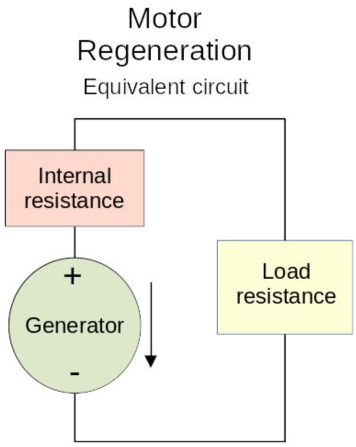
REGENERATIVE POWER THROUGH RESISTANCE

Equivalent circuit and parameters

The figure shows a simplified equivalent circuit of a motor being driven on overrun into a resistive load.

The following variables are used in the analysis (those marked * are usually specified by the supplier):

Vs	* the supply voltage
Vb	the back EMF generated when in overrun (Volts)
I	the current (Amps)
Res	the motor internal resistance (Ohms)
Rl	the resistance of a resistive load (Ohms)
R	the revolutions per minute
Rm	* the maximum (free-running) rpm
Wg	the regenerated electrical power (Watts)
Wl	power delivered into the external resistive load
T	the regenerative torque (Newton-metres)
E	the mechanical work done per revolution
η	the power efficiency



The generated voltage is the same as the back EMF for the same rotation rate

101 $Vb = Vs*R/Rm$

But his doesn’t come without a cost. The motor has an electrical resistance, so anything that attempts to derive power from the motor will suffer losses through heat generated in the resistance of the motor itself. In the extreme case, when the motor terminals are simply short-circuited, the generated current is limited only by the internal resistance, and the available power is entirely dispersed as heat within the motor.

102 $Wg = Vb^2/Res$

If the motor is being run at a large fraction of the rated speed the heat produced in the motor can approach that of the stall situation under full power, where the heat may be double the rated power of the motor. This may lead to it’s early destruction.

Available power

If the external load resistance is zero (short circuit) then, having no voltage across it, it will not provide any external power. Similarly, if a very high resistance is used (an open circuit) then there will be no current, and no external power will be available, but if something in between is used then power and energy can be harvested for external use.

The current is simply the generated voltage divided by the sum of the resistances.

103 $I = Vb/(Res+Rl)$

So the power provided by the motor, which comes from kinetic energy and which therefore provides regenerative braking, is

104 $W = I*Vb = Vb^2/(Res+Rl)$

So a higher external resistance collects less power from the motor and therefore provides a reduced regenerative braking effect.

The total power delivered into the external load is

105 $Wl = I^2*Rl = Vb^2*Rl/(Res+Rl)^2$

This shows that increasing the resistance diminishes the external power, but the effect becomes less apparent as the external resistor becomes large in comparison with the motor resistance.

Torque

This translates, through the drive gearing, to the braking effect on the vehicle. As with the motor effect, the work done per revolution is the torque multiplied by the circumference of an imaginary wheel of radius 1 metre, so this has to match the total amount of energy being collected by the motor per revolution.

106 E = T*2*pi = E*60/R

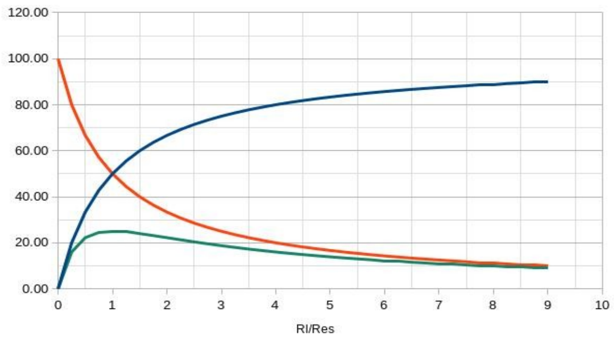
106a (rearranging) T = E*30/(R*pi)

Substituting E from 104, and Vb from 101

107 $T = Vs^2*R*30/((Res+Rl)*pi*Rm^2)$

Efficiency

The power transfer efficiency is important because it shows the proportion of lost kinetic energy that is available electrically, and is potentially returnable to the battery. So from 105 and 104



108 $\eta = Vb^2*Rl/(Res+Rl)^2/(Vb^2/(Res+Rl))$

108a (rearranging) $\eta = Rl/(Res+Rl)$

Which confirms that a high resistive load increases efficiency, and a short circuit has zero efficiency (where all the energy that is generated is dissipated within the motor itself, as discussed earlier).

The graph shows that as the external resistance becomes large in relation to the motor resistance the efficiency (shown in blue) doesn’t change much, so an external resistance of only four times the internal resistance can theoretically achieve a conversion efficiency as high as 80%.

The graph also shows (in red, compared with a short circuit) the total power recovered from kinetic energy (including that lost as heat in the motor) reduces sharply with load, even at low values.

The actual potential for recovery is the product of these two factors (from 105), which is shown (as a fraction of the total kinetic energy collected) in green. This indicates that the recoverable energy is greatest when the load resistance matches the motor resistance, but diminishes slowly with increased value.

The maximum recoverable energy occurs when the available power is split equally and 25% of the possible short-circuit braking energy is available for recovery.

Energy yield

The total power taken from the kinetic energy of the vehicle is given by

109 $Wtot = Vb^2/(Res+Rl)$

so from 101

110 $Wtot = Vs^2*(R/Rm)^2/(Res+Rl)$

So the greatest yield being at 50% conversion efficiency and when Rl=Res

111 $Wl <= Vs^2*(R/Rm)^2/(4*Res)$

Power tradeoff

The foregoing suggests that the best available energy recovery is 25% of the regenerative braking effect. However there is some flexibility as slightly increasing the external resistance would not substantially affect the efficiency while allowing a reduced braking effect if required.

But herein lies another problem: mechanical energy is also being lost through friction and windage. These will demand an unrecoverable share of the kinetic energy throughout the time the vehicle is moving. Indeed, on a level surface the vehicle will soon slow down and stop entirely on it’s own.

So to get the greatest possible recovery the following points may act as guidance:

- It is desirable to use regenerative braking wherever possible using the optimum load resistance.
- When weaker braking is required it could be achieved by using a PWM switching system, or the load resistance should be increased to harvest more of the lower energy level.
- When stronger braking is required it would be better to decrease the load resistance instead of using mechanical braking, even though that would incur lower conversion efficiency, subject to limiting the heat dissipation in the motor. This also supposes that the load resistance is in some way variable, which may present difficulties.

It should be remembered that from the foregoing equations that the total regenerative energy is proportional to the square of the voltage, and therefore the square of the speed of the vehicle, which is then proportional to it’s kinetic energy. And from 107 the regenerative torque is proportional to the revolutions which is proportional to the speed, so if all other things remain the same the braking effect will reduce with speed.

A possibility that should be considered is to disregard the popular assumption that electric motors do not benefit from gear changes. In order to increase the regenerative power a reduced gear ratio, leading to increased motor revolutions, could substantially increase the available power. For example, a factor or two in the motor speed would multiply the available power by a factor of four, which could be a valuable improvement at moderate to low speeds when energy levels are lower, and it would double the weaker braking effect at lower speeds.

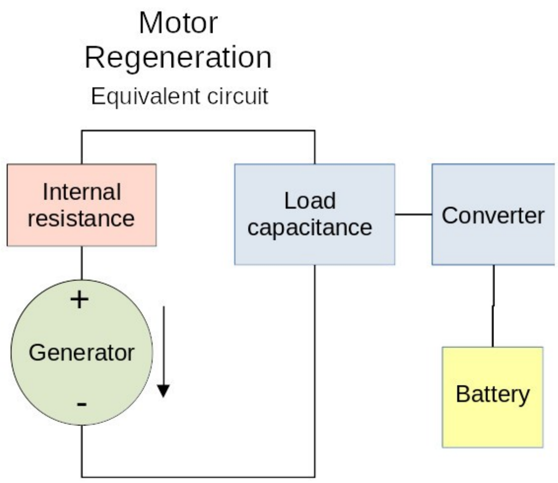
All the above assumes that the regenerative load is resistive and the power from the motor is continuous, although if a PWM control system is used to provide better driver control of braking the power would be pulsed. This creates complications for the circuit that extracts the power to the battery, and it may be difficult for such a circuit to present itself as a stable resistance.

Summary

- The available regenerated power is given in 111, limited by battery voltage and motor resistance, and is proportional to the square of the speed.
- At best, only half the regenerated power can be converted into electrical power. The rest is lost as heat in the motor.
- Optimum efficiency is achieved by the load resistance matching that of the motor.
- At optimum efficiency the total power drawn from the motor is half that of a short circuit, so the regenerative braking effect will be half that of a short circuit unless it is qualified using PWM control.
- The recovered electrical power is proportional to the square of the speed, and therefore the kinetic energy of the vehicle, and reduces substantially at slow speeds.
- The braking torque is proportional to the rotation rate, so it diminishes with speed.
- The braking effect, and corresponding braking torque can be moderated using PWM motor switching.
- It is a popular misconception that electric motors do not benefit from gearboxes. The energy recovery rate and the braking torque can be improved by changing the drive gearing to increase motor revolutions at low vehicle speeds. This could also substantially improve the energy efficiency when accelerating.
- If regenerative braking is insufficient at optimum efficiency then it should be supplemented using mechanical braking. Reducing the converter input voltage Vc will increase the braking torque but reduce the recovered power.

REGENERATIVE POWER THROUGH CAPACITANCE

An alternative approach, that may be more flexible and easier to manage, is to collect the regenerative power into a capacitance and use an electronic converter to maintain a constant voltage. The converter would then continually draw whatever current the motor provides, whether it is switched or continuous.



Equivalent circuit and parameters

The figure shows a simplified equivalent circuit of a motor being driven on overrun into a capacitive load.

The following variables are used in the analysis (those marked * are usually specified by the supplier):

Vs	* the supply voltage
Vb	the back EMF generated when in overrun (Volts)
I	the current (Amps)
Res	the motor internal resistance (Ohms)
C	the capacitance (Farads)
R	the revolutions per minute
Rm	* the maximum (free-running) rpm
Wg	the total regenerated electrical power (Watts)
Wc	power delivered into the capacitor
T	the regenerative torque (Newton-metres)
W	the mechanical work done per revolution
M	the mechanical power (work per second)
t	the time period if PWM motor control is used (seconds)
η	the power efficiency

The following conditions are assumed:

- The generated voltage is the same as the back EMF for the same rotation rate.
- This is not the same as the capacitor voltage, due to the voltage drop in the motor resistance.
- The motor output is rectified before the capacitor so it operates in either direction of rotation.
- Rectifier losses are considered to be unimportant for the initial analysis.
- The voltage on the capacitance is constant.

For a constant voltage on the capacitor the time constant of the capacitor and the motor resistance must be substantially longer that the time period of the PWM controller, if used.

201 Res*C >> t

If a switching circuit is used in the converter, the same rule applies to the time period of that circuit.

While these two switching circuits may operate asynchronously, the net effect is that the converter receives an average current that matches that from the motor at all times, and doesn't vary significantly other than by following the motor rotation rate.

The current flowing into the capacitor depends on the generated voltage, the voltage on the capacitor and the resistance of the motor

201 I = (Vb-Vc)/Res

This matches the current through the converter, so the effective available power through the converter is

202 Wc = Vc*I = Vc*(Vb-Vc)/Res

And the total regenerated power is

203 $Wg = Vb*(Vb-Vc)/Res$

This suggests that zero converter power occurs when either Vc is at zero or Vc is allowed to drift up to the generator voltage. So, as with the resistive load, there has to be an optimum value for Vc as a fraction of Vb. But Vb varies continually with motor speed.

204 $Vb = Vs*R/Rm$

So this time we are looking to optimise a variable operating voltage for the capacitor and converter, where previously we needed to optimise the fixed load resistance.

The energy relationship from 202 is an inverted quadratic (hump upwards) with zero values at Vc=0 or Vc=Vb. The highest value is halfway between these points, so for the maximum power conversion

205 $Vc=Vb/2$

Conversion efficiency

The power conversion efficiency is the ratio of converter power to total regenerated power

206 $\eta = Vc*I/(Vb*I) = Vc/Vb$

So from 205 the best conversion efficiency achievable is

207 $\eta = 50\%$

Energy yield

The total power taken from the kinetic energy of the vehicle is given by

208 $Wtot = Vb^2/Res$

so from 204

209 $Wtot = Vs^2*(R/Rm)^2/Res$

So the greatest yield being at 50% conversion efficiency

210 $Wc <= Vs^2*(R/Rm)^2/(2*Res)$

Conversion algorithm

But Vb is a notional internal voltage that is not available at the motor terminals, so it somehow has to be derived. From 201 the current can be found for the optimum Vc and a known value for Res (this can be measured or calculated from the earlier section).

211 $I = (2Vc-Vc)/Res = Vc/Res$

So the optimum balance between current and voltage at the converter can be found

212 $Vc/I = Res$

The converter therefore needs to continually balance it’s current demand in order to sustain this relationship.

Torque

The braking torque can be calculated from the mechanical energy. From 106a we have the torque in terms of the energy per second, and we can match this with the electrical energy from 203.

213 $T = E*30/(R*pi) = Vb*(Vb-Vc)*30/(R*pi*Res)$

But from 205 and 204

214 $Vc = Vs*R/(2*Rm)$ and $Vb=2*Vc$

So

215 $T = Vc^2*4*30/(R*pi*Res) = Vs^2*R^2*120/(R*pi*Res*4*Rm^2)$

215a (rearranging) $T = Vs^2*R*30/(pi*Res*Rm^2)$

Summary

- The available regenerated power is given in 210, limited by battery voltage and motor resistance, and is proportional to the square of the speed.
- At best, only half the regenerated power can be converted into electrical power. The rest is lost as heat in the motor.
- Optimum efficiency is achieved by the converter maintaining the electrical relationship of 106.
- At optimum efficiency the total power drawn from the motor is half that of a short circuit, so the regenerative braking effect will be half that of a short circuit unless it is qualified using PWM control.
- The recovered electrical power is proportional to the square of the speed, and therefore the kinetic energy of the vehicle, and reduces substantially at slow speeds.
- The braking torque is proportional to the rotation rate, so it diminishes with speed.
- The braking effect, and corresponding braking torque can be moderated using PWM motor switching.
- It is a popular misconception that electric motors do not benefit from gearboxes. The energy recovery rate and the braking torque can be improved by changing the drive gearing to increase motor revolutions at low vehicle speeds. This could also substantially improve the energy efficiency when accelerating.
- If regenerative braking is insufficient at optimum efficiency then it should be supplemented using mechanical braking. Reducing the converter voltage Vc will increase the braking torque but reduce the recovered power.

CHARGING THE BATTERY

It is apparent from the above that under normal conditions the regenerated voltage is a fraction of the motor rated voltage, and is therefore lower than (perhaps quite small fraction of) the battery voltage.

You may think that charging a battery from a lower voltage is like pushing water uphill. But Archimedes solved the water problem a long time ago, and since high efficiency solid-state switches (MOSFETs) have become commonplace the charging problem is now also routine.

Boost circuits

The principle of a so-called “boost” converter of a type now commonly used to produce a current from a low voltage source into a higher voltage load is explained very well in Wikipedia and needs no further explanation here.

https://en.wikipedia.org/wiki/Boost_converter